1. Rates

Interpretations of Interest Rates
- Required return, Discount rate, Opportunity cost

Required Return Decomposition
- $r = r_{\text{RF}} + \pi + \text{DRP} + \text{LP} + \text{MP}$
- $r_{\text{RF}} = r_{\text{real}} + \pi$

Future / Present Value
- $FV = PV(1+r)^n$, continuous: $FV = PVe^{rn}$
- $PV = \frac{FV}{(1+r)^n}$, continuous: $PV = FVe^{-rn}$
- *Discount factor*: $DF = (1+r)^{-n}$ (= principal value factor)

Non-Annual Compounding
- $FV = PV\left(1+\frac{r_s}{m}\right)^{mN}$
- $r_{\text{eff}} = \left(1+\frac{r_s}{m}\right)^m - 1$ (EAR)
- $r_{\text{cont}} = \ln(1+r_{\text{eff}}), \quad r_{\text{eff}} = e^{r_{\text{cont}}} - 1$

Rate Conversions (exam favourites)
- BEY $\to$ EAR: EAR $= \left(1+\frac{\text{BEY}}{2}\right)^2 - 1$ (semi-annual)
- MMY $\approx$ BEY $\times \frac{360}{\text{days to mat.}}$

Annuities & Perpetuities
- Ordinary annuity: $PV = A\,\frac{1-(1+r)^{-n}}{r}$
- Annuity due: $PV_{\text{due}} = PV_{\text{ord}}(1+r)$
- Perpetuity: $PV = \frac{PMT}{r}$, Growing perpetuity: $PV = \frac{CF_1}{r-g}$

Bond Pricing & YTM Algebra
- $P = \sum_{t=1}^{N} \frac{C}{(1+Y)^t} + \frac{M}{(1+Y)^N}$ solve $Y \Rightarrow$ YTM

2. Return Measures

- Holding-period: $R = \frac{P_1 - P_0 + CF}{P_0}$
- Multi-period: $R_{0,T} = \prod_{t=1}^{T}(1+R_t) - 1$
- Arithmetic: $\bar{R} = \frac{1}{n}\sum R_i$; Geometric: $R_G = \left(\prod(1+R_i)\right)^{1/n} - 1$; Harmonic: $R_H = \frac{n}{\sum \frac{1}{R_i}}$
- Inequality: $R_H \leq R_G \leq \bar{R}$ (volatile returns)
- IRR: $\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$; Time-weighted: geometric link of sub-period returns
- *Real return*: $r_{\text{real}} \approx \frac{1+r_{\text{nom}}}{1+\pi} - 1$

3. Descriptive Statistics

- Location: mean, median, mode Dispersion: range, s^2, s, CV$=s/\bar{x}$
- Skew: $g_1 = \frac{\sum(x-\bar{x})^3}{ns^3}$; Kurtosis: $g_2 = \frac{\sum(x-\bar{x})^4}{ns^4}$
- Empirical rule (normal): 68-95-99.7 % Chebyshev: $\geq 1 - \frac{1}{k^2}$ within $k\sigma$
- Critical $|g_1| > 2\sqrt{\frac{6}{n}}$ skewed; $|g_2| > 2\sqrt{\frac{24}{n}}$ leptokurtic
- Downside deviation: SD of observations $< B$

4. Probability Concepts & Counting

- $P(A \cup B) = P(A) + P(B) - P(A \cap B), \quad P(A \cap B) = P(A|B)P(B)$
- Total prob: $P(A) = \sum_i P(A|S_i)P(S_i)$; Bayes: $P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
- Expectation: $E(X) = \sum x_i P(x_i)$; $\text{Var}(X) = E[(X-\mu)^2]$
- Combinatorics: ${}_nP_r = \frac{n!}{(n-r)!}$, ${}_nC_r = \frac{n!}{r!(n-r)!}$

5. Portfolio Mathematics

- $E(R_p) = \sum w_i E(R_i), \quad \sigma_p^2 = \sum\sum w_i w_j \text{Cov}(R_i, R_j)$
- Beta: $\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$; CAPM: $E(R_i) = r_f + \beta_i(E(R_M) - r_f)$
- Min-var (2 assets): $w^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$
- Corr: $\rho_{ij} = \frac{\text{Cov}(R_i, R_j)}{\sigma_i \sigma_j}$; Safety-first: $SF = \frac{E(R_p) - R_L}{\sigma_p}$

6. Distributions

- Normal: $z = \frac{x-\mu}{\sigma}$; Lognormal: $\ln S \sim N(\mu, \sigma^2)$
- Continuous compounding: $S_T = S_0 e^{rT}$
- Monte-Carlo (stochastic model) vs Historical (resampling); RNG pitfalls: serial correlation, seed bias
- Random walk: $X_t = X_{t-1} + \varepsilon_t$ (unit root); AR(1): $X_t = \phi X_{t-1} + \varepsilon_t$, stationary if $|\phi| < 1$

7. Sampling & Estimation

- CLT: $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$ for $n \geq 30$ SE: $s/\sqrt{n}$
- Finite-pop correction: $SE_{\text{fpc}} = \frac{s}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$
- CI (known σ): $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ unknown: replace with $t_{\alpha/2,df}$
- Jackknife (leave-one-out), Bootstrap (resample B times)

8. Hypothesis Testing

- $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}, \quad t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ (df $= n-1$)
- $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}, \quad F = \frac{s_1^2}{s_2^2}$
- Critical z: 1.645 (90%), 1.96 (95%), 2.576 (99%)
- p-value = smallest α s.t. reject H_0; Type I/II, power $= 1 - \beta$

9. Non-Parametric & Independence Tests

- Sign, Wilcoxon signed-rank (paired), Mann-Whitney U (indep.), Kruskal-Wallis (k samples)
- Spearman: $r_S = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$
- Chi-square contingency: $\chi^2 = \sum \frac{(O-E)^2}{E}$, Cramér's $V = \sqrt{\frac{\chi^2}{n(k-1)}}$

10. Simple Linear Regression

- $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i, \quad b_1 = \frac{\text{Cov}(X,Y)}{\text{Var}(X)}, \quad b_0 = \bar{Y} - b_1\bar{X}$
- ANOVA: $SST = SSR + SSE$, $R^2 = SSR/SST$, SEE $= \sqrt{MSE}$
- $t_{b_i} = \frac{b_i - \beta_{i,0}}{SE(b_i)}$, Prediction: $\hat{Y}_0 \pm t_{\alpha/2} SE_{\text{pred}}$
- Durbin-Watson: $DW = \frac{\sum(e_t - e_{t-1})^2}{\sum e_t^2}$ ($\approx$ 2 means no autocorr.)
- Std. beta: $\beta_{\text{std}} = \beta_1 \frac{\sigma_X}{\sigma_Y}$ (unit-free magnitude)

11. Duration

- Macaulay: $D_M = \frac{\sum t\, PV(CF_t)}{P}$, Modified: $D_{\text{mod}} = \frac{D_M}{1+Y/m}$
- Convexity: $C = \frac{\sum t(t+1)PV(CF_t)}{P(1+Y)^2}$, $\%\Delta P \approx -D_{\text{mod}}\Delta Y + 0.5C\Delta Y^2$
- Decision tree: EV$= \sum p_i CF_i$ at each node, discount back with risk-adj. rate

12. FinTech & Big Data

- 3Vs + veracity, value; ML: supervised, unsupervised, reinforcement, deep
- Overfitting vs underfitting; train/validation/test split
- Alternative data sources; pipeline: ingest $\to$ clean $\to$ feature $\to$ model

Tulga-Ochir Sugar — personal cheat-sheet Quant (not exhaustive)